A measurement of pulse separations for photoelectrons in the case of a Lorentz spectral line

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A measurement of pulse separations for photoelectrons in the case of a Lorentz spectral line†

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Abstract. Measurements of the width of a Lorentz spectral line using a digital photoelectron correlation device are compared with measurements made on the same gaussian light source by a pulse separation detector. The agreement of the two measurements over a wide range of count rates provides a test of recent theories of pulse separation measurements.

Early measurements of spectral linewidths using photon correlation techniques involved devices which determine coincidence rates between photons detected during two time intervals separated by a fixed time (Hanbury-Brown and Twiss 1954, Rebka and Pound 1957). Recently, pulse autocorrelators have been introduced which measure joint counting rates over a range of delay times simultaneously (Jakeman *et al.* 1968, Chen and Polonsky-Ostrowsky 1969).

Equivalent information can be obtained from experiments which simply measure the distribution of separation times between individual photons. The width of a mercury vapour line was measured with such a device by Scarl (1968). In his experiment the count rates were so small that the probability of detecting two consecutive photons separated by a time T (which we shall call $P(T)$) could be approximated by the joint counting rate at time intervals centred at $t = 0$ and $t = T$. This approximation is good only when the probability of measuring a count during the time interval T is small (Glauber 1968, Barakat and Glauber 1971). Another approximation, good when T is small compared with a coherence time, has been used by Bendjaballah and Perrot (1971) to study nonthermal gaussian fields. In this letter we use the exact analysis of pulse separation statistics introduced by Glauber to enable us to interpret experiments in which these approximations are no longer valid.

Using standard photon counting techniques it can be shown that

$$P(T)\,\mathrm{d}T = w\left.\frac{\partial^2 Q(s, T)}{\partial T^2}\right|_{s=1} \tag{1}$$

where w is the average count rate and $Q(s, T)$ is the single time generating function defined by

$$Q(s, T) = \sum_{n=0}^{\infty} (1-s)^n P(n, T) \tag{2}$$

and $P(n, T)$ is the probability of measuring n photons during a time T.

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When the incident light is gaussian, has a Lorentz spectrum of width Γ, and the detector is small compared with a coherence area, then $Q(s, T)$ and $P(t)$ can be calculated exactly (Jakeman and Pike 1968)

$$Q(s, T) = \frac{4\,Y(s)\,e^{\tau}}{(1+Y(s))^2 \exp(Y(s)\tau) - (1-Y(s))^2 \exp(-Y(s)\tau)} \tag{3}$$

where

$$\tau = T\Gamma$$

$$v = \frac{w}{\Gamma}$$

$$Y(s) = (1+2vs)^{1/2}.$$

Corrections to (3) for detectors of finite size have also been obtained (Kelly 1972a, Jakeman *et al* 1970).

In the limiting case where $\tau v \ll 1$, we can use equations (1) and (3) to show that

$$P(T) = w(1+e^{-2\tau}). \tag{4}$$

Equation (4) also describes the joint counting rate at time intervals centred at $t = 0$ and $t = T$, provided that each interval is short compared with $1/\Gamma$. This is the functional form used by Scarl.

In a previous paper (Kelly and Blake 1971) we have provided a direct experimental verification of (3) over a large range of v and τ. Here we extend that verification by using relation (1).

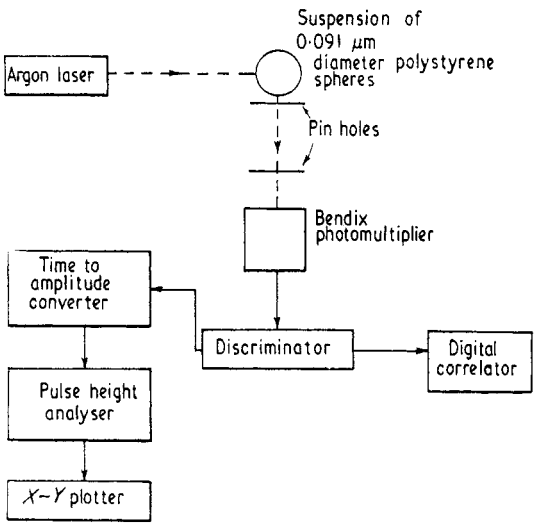

Figure 1. Apparatus for measuring pulse separations.

In our experiments (see figure 1) we measure pulse separations by sending the output of a photomultiplier (Bendix 754) through a discriminator (EG and G TD101/N in LLT mode) and into a time to amplitude converter (EG and G TH200A/N modified to increase its full scale time). The output pulses from the time to amplitude converter are sorted and recorded by a pulse height analyser (HP 5400 MCA in PHA mode). The light illuminating the photomultiplier was produced by scattering coherent laser light

with a wavelength of 4880 Å from an aqueous suspension of polystyrene spheres 0·091 μm in diameter. This produced light with a Lorentz spectrum, the linewidth $\Gamma = (3{\cdot}52 \pm 0{\cdot}05) \times 10^3\ s^{-1}$. Strictly speaking, this light is 'pseudogaussian' due to the finite coherence time of the incident field, but the photocounting statistics will be the same as those for a true gaussian field (Picinbono and Rosseau 1970).

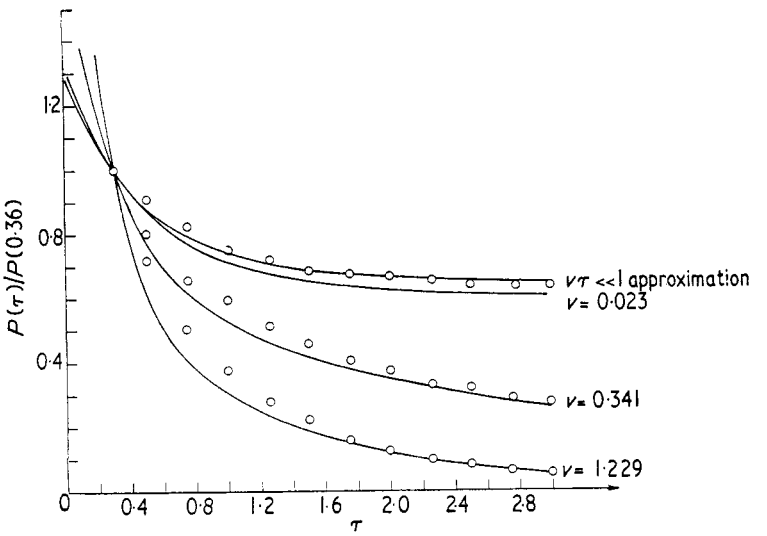

Figure 2. Theoretical (full curves) and experimental (open circles) plots of $P(\tau)$ against τ.

The results of the experiments are shown in figure 2. The full curves, calculated by using equations (1) and (3), show good agreement with our measurements. We believe that the departure at high count rates and small τ is probably due to the finite pulse widths used by our equipment, and to the necessity of providing a 'dead time' while our equipment analyses each event. These results indicate that pulse separation measurements can yield information similar to the autocorrelation measurements if the results are correctly interpreted (Kelly 1972b).

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K shell electron wavefunctions in complex atoms

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Abstract. A simple procedure for obtaining 1s electron wavefunctions in complex atoms is presented which takes into account many body, relativistic and finite nuclear size effects.

In recent years there have been a number of solutions to the Hartree–Fock problem in complex atoms. Perhaps the most widely used solutions are those given by Herman and Skillman (1963, to be referred to as HS). HS give numerical solutions to the Hartree–Fock equations for most atoms in the periodic table. These solutions are obtained in the Slater free electron approximation (Slater 1960). Finite nuclear size effects are ignored and relativistic corrections to the single particle energies are obtained in first order perturbation theory.

In heavy atoms the inner shell electron wavefunctions (in particular those of the 1s shell) may be substantially modified by the finite nuclear size and relativistic effects. These modifications are of significance in the calculation of electron capture cross sections. We report here on our results for K shell electrons based upon the HS compilations.

Suppose H is the full relativistic single particle Hamiltonian for the 1s electronic state Ψ. Then Ψ is the proper eigenfunction to use in calculating electron capture cross sections. The Hamiltonian H_0 and eigenfunctions Ψ_0 of the relativistic hydrogenic problem (nucleus of charge Z) are well known. We may construct a Foldy–Wouthuysen (FW) transformation F_0 (Rose 1961) such that Φ_0 is the classical limit of Ψ_0

$$\Phi_0 = F_0\Psi_0. \tag{1}$$

Similarly we can define F such that Φ is the classical limit of Ψ. Then if

$$\Phi = \mathscr{A}\Phi_0 \tag{2}$$